Formulas for mathematics 4

Algebra

Rules

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Quadratic equations

$x^2 + px + q = 0$

$x = -\dfrac{p}{2} \pm \sqrt{\left(\dfrac{p}{2}\right)^2 - q}$

$ax^2 + bx + c = 0$

$x = -\dfrac{b}{2a} \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$

Arithmetic

Prefixes

T	G	M	k	h	d	c	m	μ	n	p
tera	giga	mega	kilo	hecto	deci	centi	milli	micro	nano	pico
10^{12}	10^{9}	10^{6}	10^{3}	10^{2}	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}

Powers

$a^x a^y = a^{x+y}$ $\qquad \dfrac{a^x}{a^y} = a^{x-y}$ $\qquad (a^x)^y = a^{xy}$ $\qquad a^{-x} = \dfrac{1}{a^x}$

$a^x b^x = (ab)^x$ $\qquad \dfrac{a^x}{b^x} = \left(\dfrac{a}{b}\right)^x$ $\qquad a^{\frac{1}{n}} = \sqrt[n]{a}$ $\qquad a^0 = 1$

Geometric series

$a + ak + ak^2 + \ldots + ak^{n-1} = \dfrac{a(k^n - 1)}{k - 1}$ where $k \neq 1$

Logarithms

$y = 10^x \Leftrightarrow x = \lg y$ $\qquad y = \mathrm{e}^x \Leftrightarrow x = \ln y$

$\lg x + \lg y = \lg xy$ $\qquad \lg x - \lg y = \lg \dfrac{x}{y}$ $\qquad \lg x^p = p \cdot \lg x$

Absolute value

$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$

Functions

Linear function

$y = kx + m$ $\quad k = \dfrac{y_2 - y_1}{x_2 - x_1}$

$ax + by + c = 0$, where a and b are not both zero

Quadratic function

$y = ax^2 + bx + c \quad a \neq 0$

Power function

$y = C \cdot x^a$

Exponential function

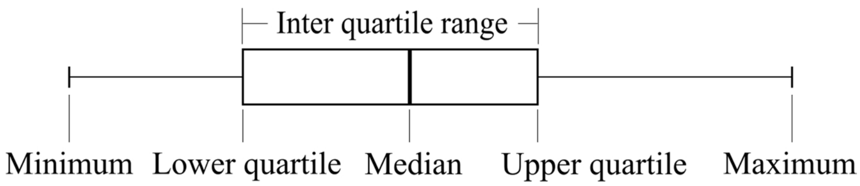
$y = C \cdot a^x \quad a > 0$ och $a \neq 1$

Statistics and probability

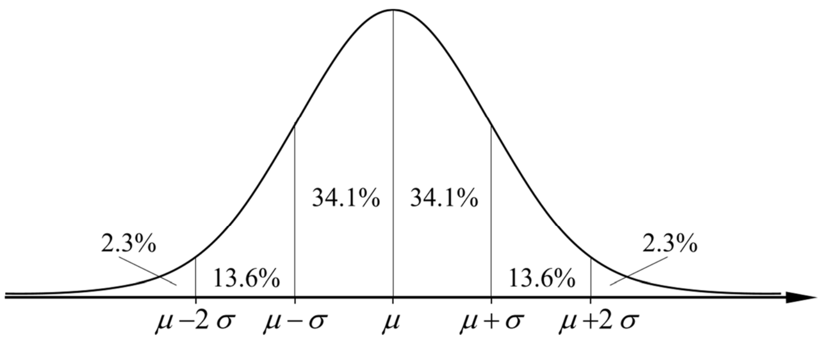
Standard deviation of a sample

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n - 1}}$$

Box plot



Normal distribution



Density function of the normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Differential and integral calculus

Definition of the derivative

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Derivative

Function	Derivative
x^n where n is a real number	nx^{n-1}
$a^x \quad (a > 0)$	$a^x \ln a$
$\ln x \; (x > 0)$	$\frac{1}{x}$
e^x	e^x
e^{kx}	$k \cdot e^{kx}$
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$1 + \tan^2 x = \frac{1}{\cos^2 x}$
$k \cdot f(x)$	$k \cdot f'(x)$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$\frac{f(x)}{g(x)} \quad (g(x) \neq 0)$	$\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$

Chain rule

If $y = f(z)$ and $z = g(x)$ are two differentiable functions then it holds for $y = f(g(x))$ that

$y' = f'(g(x)) \cdot g'(x)$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}z} \cdot \frac{\mathrm{d}z}{\mathrm{d}x}$

Antiderivatives

Function	Antiderivatives
k	$kx + C$
$x^n \quad (n \neq -1)$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln x + C \quad (x > 0)$
e^x	$e^x + C$
e^{kx}	$\frac{e^{kx}}{k} + C$
$a^x \quad (a > 0,\ a \neq 1)$	$\frac{a^x}{\ln a} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$

Complex numbers

Representation $z = x + iy = re^{iv} = r(\cos v + i\sin v)$ where $i^2 = -1$

Argument $\arg z = v \qquad \tan v = \frac{y}{x}$

Absolute value $|z| = r = \sqrt{x^2 + y^2}$

Conjugate If $z = x + iy$ then $\bar{z} = x - iy$

Rules $z_1 z_2 = r_1 r_2 (\cos(v_1 + v_2) + i\sin(v_1 + v_2))$

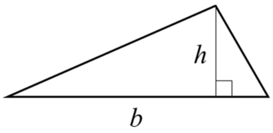
$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(v_1 - v_2) + i\sin(v_1 - v_2))$$

de Moivre's formula $z^n = (r(\cos v + i\sin v))^n = r^n(\cos nv + i\sin nv)$

Geometry

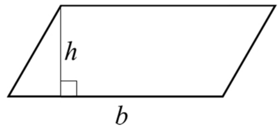
Triangle

$$A = \frac{bh}{2}$$



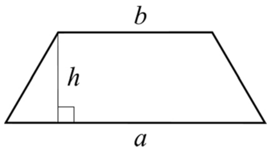
Parallelogram

$$A = bh$$



Trapezium

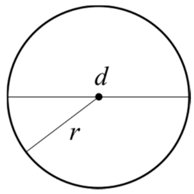
$$A = \frac{h(a+b)}{2}$$



Circle

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

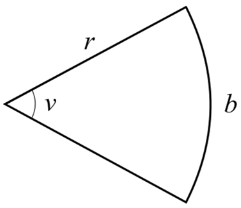
$$O = 2\pi r = \pi d$$



Circle sector

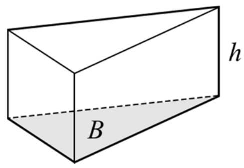
$$b = \frac{v}{360°} \cdot 2\pi r$$

$$A = \frac{v}{360°} \cdot \pi r^2 = \frac{br}{2}$$



Prism

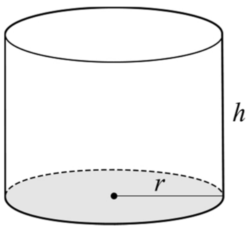
$$V = Bh$$



Cylinder

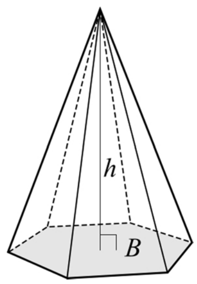
$$V = \pi r^2 h$$

Lateral surface area
$$A = 2\pi rh$$



Pyramid

$$V = \frac{Bh}{3}$$

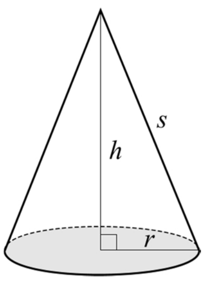


Cone

$$V = \frac{\pi r^2 h}{3}$$

Lateral surface area

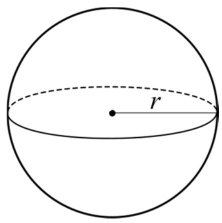
$$A = \pi rs$$



Sphere

$$V = \frac{4\pi r^3}{3}$$

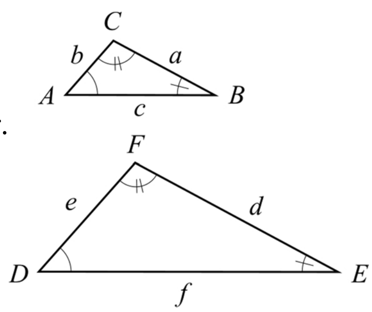
$$A = 4\pi r^2$$



Similarity

The triangles *ABC* and *DEF* are similar.

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$



Scale

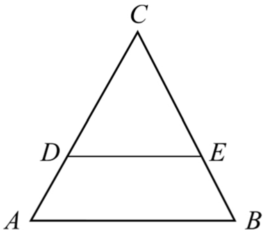
Area scale factor = (Length scale factor)^2
Volume scale factor = (Length scale factor)^3

Triangle with a transversal line

If DE is parallel to AB then

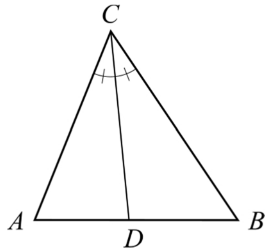
$$\frac{DE}{AB} = \frac{CD}{AC} = \frac{CE}{BC} \text{ and}$$

$$\frac{CD}{AD} = \frac{CE}{BE}$$



Angle bisector theorem

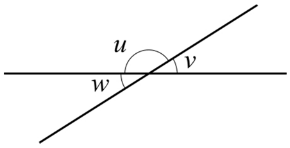
$$\frac{AD}{BD} = \frac{AC}{BC}$$



Angles

$u + v = 180°$ Supplementary angles

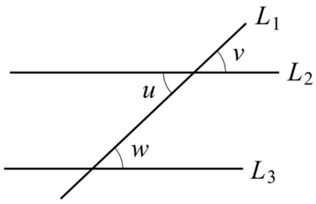
$w = v$ Vertical angles



L_1 cuts two parallel lines L_2 and L_3

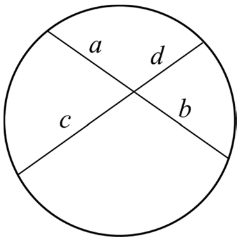
$v = w$ Corresponding angles

$u = w$ Alternate angles



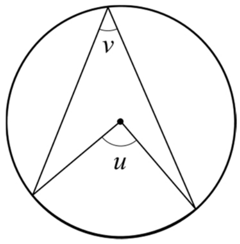
Chord theorem

$ab = cd$



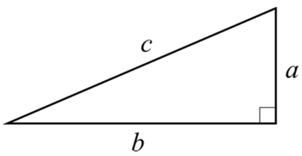
Angles subtended by the same arc

$u = 2v$



Pythagoras' theorem

$a^2 + b^2 = c^2$



Distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint formula

$$x_m = \frac{x_1 + x_2}{2} \text{ and } y_m = \frac{y_1 + y_2}{2}$$

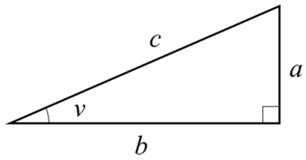
Trigonometry

Definitions

$$\sin v = \frac{a}{c}$$

$$\cos v = \frac{b}{c}$$

$$\tan v = \frac{a}{b}$$

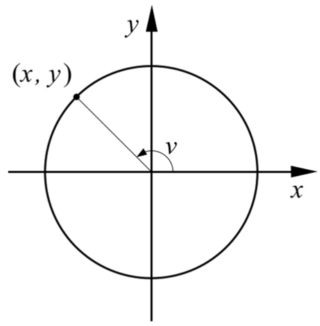


Unit circle

$$\sin v = y$$

$$\cos v = x$$

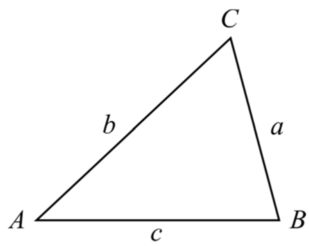
$$\tan v = \frac{y}{x}$$



Sine rule $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine rule $$a^2 = b^2 + c^2 - 2bc\cos A$$

Area $$T = \frac{ab\sin C}{2}$$



Trigonometric formulas

$$\sin^2 v + \cos^2 v = 1$$

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\sin 2v = 2\sin v \cos v$$

$$\cos 2v = \begin{cases} \cos^2 v - \sin^2 v & (1) \\ 2\cos^2 v - 1 & (2) \\ 1 - 2\sin^2 v & (3) \end{cases}$$

$$a\sin x + b\cos x = c\sin(x+v) \text{ where } c = \sqrt{a^2+b^2} \text{ and } \tan v = \frac{b}{a}$$

Circle equation $$(x-a)^2 + (y-b)^2 = r^2$$

Exact values

Angle v (degrees) (radians)	$0°$ 0	$30°$ $\frac{\pi}{6}$	$45°$ $\frac{\pi}{4}$	$60°$ $\frac{\pi}{3}$	$90°$ $\frac{\pi}{2}$	$120°$ $\frac{2\pi}{3}$	$135°$ $\frac{3\pi}{4}$	$150°$ $\frac{5\pi}{6}$	$180°$ π
$\sin v$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan v$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not def.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0