# Formulas for mathematics 4

## Algebra

**Rules**

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

**Quadratic equations**

$x^2 + px + q = 0$ $\qquad$ $ax^2 + bx + c = 0$

$x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$ $\qquad$ $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

## Arithmetic

**Prefixes**

| T | G | M | k | h | d | c | m | μ | n | p |
|---|---|---|---|---|---|---|---|---|---|---|
| tera | giga | mega | kilo | hecto | deci | centi | milli | micro | nano | pico |
| $10^{12}$ | $10^{9}$ | $10^{6}$ | $10^{3}$ | $10^{2}$ | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ | $10^{-6}$ | $10^{-9}$ | $10^{-12}$ |

**Powers**

$a^x a^y = a^{x+y}$ $\qquad$ $\frac{a^x}{a^y} = a^{x-y}$ $\qquad$ $(a^x)^y = a^{xy}$ $\qquad$ $a^{-x} = \frac{1}{a^x}$

$a^x b^x = (ab)^x$ $\qquad$ $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$ $\qquad$ $a^{\frac{1}{n}} = \sqrt[n]{a}$ $\qquad$ $a^0 = 1$

**Geometric series**

$a + ak + ak^2 + \ldots + ak^{n-1} = \frac{a(k^n - 1)}{k-1}$ where $k \neq 1$

**Logarithms**

$y = 10^x \Leftrightarrow x = \lg y$ $\qquad$ $y = e^x \Leftrightarrow x = \ln y$

$\lg x + \lg y = \lg xy$ $\qquad$ $\lg x - \lg y = \lg \frac{x}{y}$ $\qquad$ $\lg x^p = p \cdot \lg x$

**Absolute value**

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

# Functions and relations

### Linear function

$y = kx + m$ $\qquad k = \dfrac{y_2 - y_1}{x_2 - x_1}$

$k_1 \cdot k_2 = -1$, condition for perpendicular lines

$ax + by + c = 0$, where $a$ and $b$ are not both zero

### Quadratic functions

$y = ax^2 + bx + c$ $\qquad a \neq 0$

### Power functions

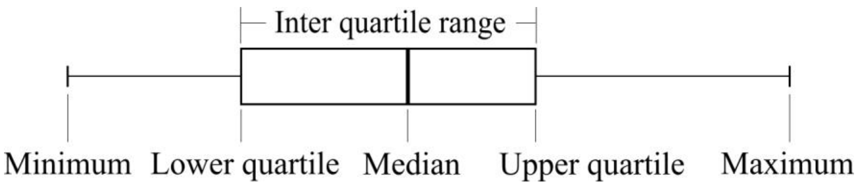
$y = C \cdot x^a$

### Exponential functions

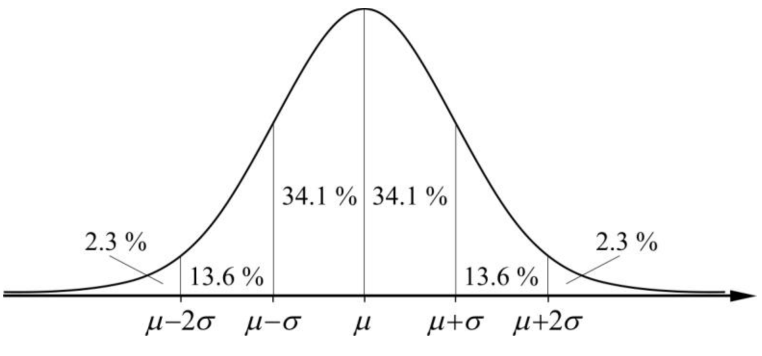
$y = C \cdot a^x$ $\qquad a > 0$ and $a \neq 1$

# Statistics and probability

### Box plot




### Normal distribution




### Density function of the normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

# Differential and integral calculus

**Definition of the derivative**

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

**Derivatives**

| Function | Derivative |
|---|---|
| $x^n$ where $n$ is a real number | $nx^{n-1}$ |
| $\frac{1}{x}$ | $-\frac{1}{x^2}$ |
| $\ln x \quad (x > 0)$ | $\frac{1}{x}$ |
| $a^x \quad (a>0)$ | $a^x \ln a$ |
| $e^x$ | $e^x$ |
| $e^{kx}$ | $k \cdot e^{kx}$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $1 + \tan^2 x = \frac{1}{\cos^2 x}$ |
| $k \cdot f(x)$ | $k \cdot f'(x)$ |
| $f(x) + g(x)$ | $f'(x) + g'(x)$ |
| $f(x) \cdot g(x)$ | $f'(x) \cdot g(x) + f(x) \cdot g'(x)$ |
| $\frac{f(x)}{g(x)} \quad (g(x) \neq 0)$ | $\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$ |

**Chain rule**

If $y = f(z)$ and $z = g(x)$ are two differentiable functions then it holds for $y = f(g(x))$ that

$$y' = f'(g(x)) \cdot g'(x) \text{ or } \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$$

**Fundamental theorem of calculus**

$$\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a) \quad \text{where } F'(x) = f(x)$$

**Antiderivatives**

| Function | Antiderivatives |
|---|---|
| $k$ | $kx + C$ |
| $x^n \quad (n \neq -1)$ | $\frac{x^{n+1}}{n+1} + C$ |
| $\frac{1}{x}$ | $\ln x + C \quad (x > 0)$ |
| $a^x \quad (a > 0,\ a \neq 1)$ | $\frac{a^x}{\ln a} + C$ |
| $e^x$ | $e^x + C$ |
| $e^{kx}$ | $\frac{e^{kx}}{k} + C$ |
| $\sin x$ | $-\cos x + C$ |
| $\cos x$ | $\sin x + C$ |

**Volume of solids of revolution**

$\pi \cdot \int_a^b y^2\,dx$ rotation around the $x$-axis

$\pi \cdot \int_a^b x^2\,dy$ rotation around the $y$-axis

# Complex numbers

**Representation**

$z = a + bi$ Rectangular form

$z = r(\cos v + i\sin v)$ Polar form

$z = re^{iv}$ Exponential form

**Argument**

$\arg z = v \qquad \tan v = \frac{b}{a}$

**Absolute value**

$|z| = r = \sqrt{a^2 + b^2}$

**Conjugate**

$\bar{z} = a - bi$

**Rules**

$$z_1 \cdot z_2 = r_1 \cdot r_2 (\cos(v_1 + v_2) + i\sin(v_1 + v_2))$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(v_1 - v_2) + i\sin(v_1 - v_2))$$

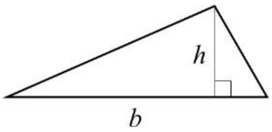
**de Moivre’s formula**

$$z^n = (r(\cos v + i\sin v))^n = r^n(\cos nv + i\sin nv)$$

# Geometry

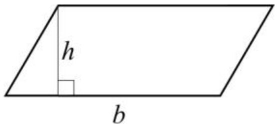
### Triangle

$$A = \frac{bh}{2}$$



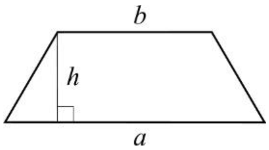

### Parallelogram

$$A = bh$$




### Trapezium

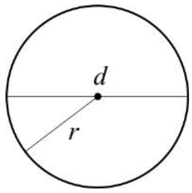
$$A = \frac{h(a+b)}{2}$$




### Circle

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

$$O = 2\pi r = \pi d$$




### Circle sector

*v* is measured in degrees

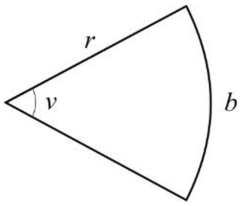
$$b = \frac{v}{360°} \cdot 2\pi r$$

$$A = \frac{v}{360°} \cdot \pi r^2 = \frac{br}{2}$$

*v* is measured in radians

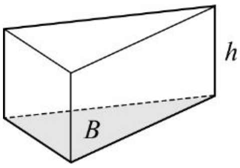
$$b = vr$$

$$A = \frac{vr^2}{2} = \frac{br}{2}$$




### Prism

$$V = Bh$$

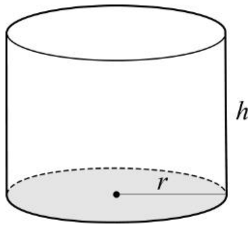



### Cylinder

$$V = \pi r^2 h$$

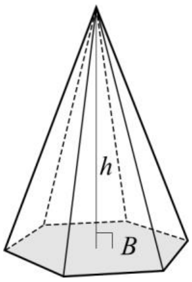
Lateral surface area

$$A = 2\pi rh$$




### Pyramid

$$V = \frac{Bh}{3}$$

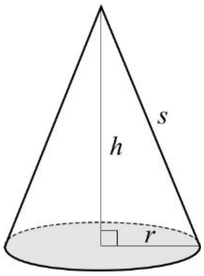



### Cone

$$V = \frac{\pi r^2 h}{3}$$

Lateral surface area

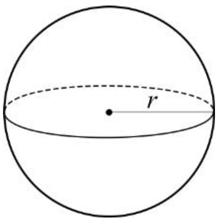
$$A = \pi rs$$




### Sphere

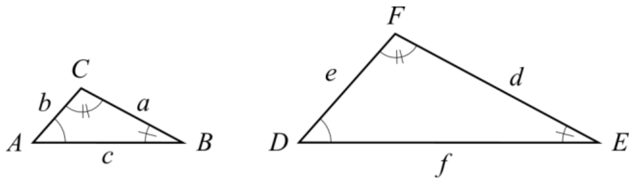
$$V = \frac{4\pi r^3}{3}$$

$$A = 4\pi r^2$$



**Similarity**

The triangles $ABC$ and $DEF$ are similar if $\dfrac{a}{d}=\dfrac{b}{e}=\dfrac{c}{f}$

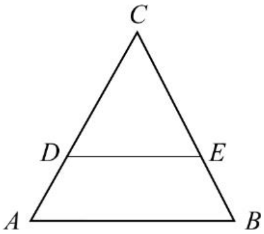



**Scale** Area scale factor = (Length scale factor)$^2$ Volume scale factor = (Length scale factor)$^3$

**Triangle with a transversal line**

$\dfrac{DE}{AB}=\dfrac{CD}{AC}=\dfrac{CE}{BC}$ and

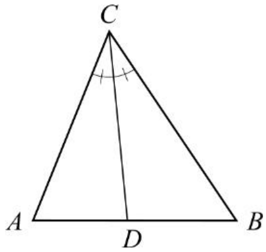
$\dfrac{CD}{AD}=\dfrac{CE}{BE}$




$DE$ is parallel to $AB$

**Angle bisector theorem**

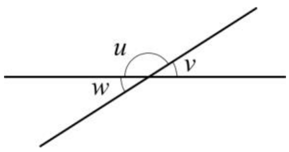
$\dfrac{AD}{BD}=\dfrac{AC}{BC}$




**Angles**

$u+v=180°$ Supplementary angles

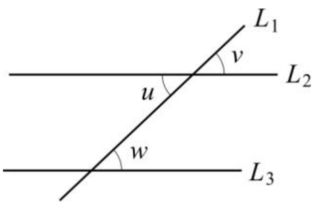
$w=v$ Vertical angles




$L_1$ intersects two parallel lines $L_2$ and $L_3$

$v=w$ Corresponding angles

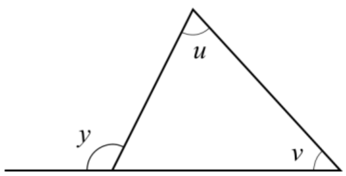
$u=w$ Alternate angles




The sum $S$ of all angles of a $n$-polygon: $S=(n-2)\cdot 180°$

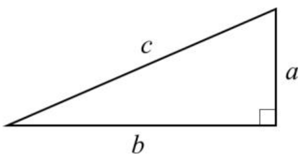
**Exterior angle theorem**

$y=u+v$



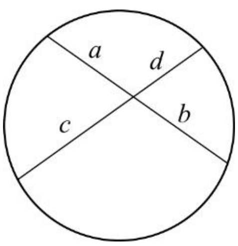

**Pythagoras' theorem**

$a^2+b^2=c^2$



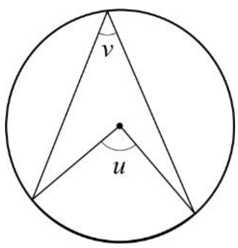

**Chord theorem**

$ab=cd$




**Angles subtended by the same arc**

$u=2v$




**Distance formula**

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

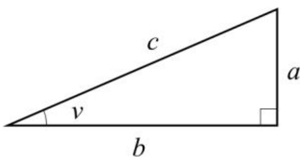
**Midpoint formula**

$$x_m=\frac{x_1+x_2}{2} \text{ and } y_m=\frac{y_1+y_2}{2}$$

# Trigonometry

**Definitions**

Right-angled triangle

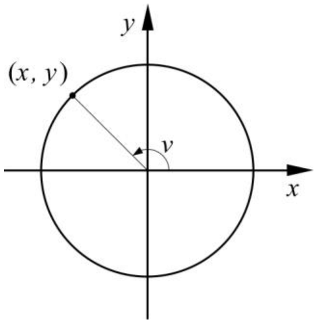



$\sin v = \dfrac{a}{c}$

$\cos v = \dfrac{b}{c}$

$\tan v = \dfrac{a}{b}$

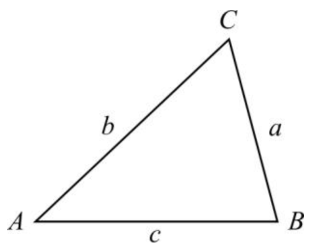
Unit circle




$\sin v = y$

$\cos v = x$

$\tan v = \dfrac{y}{x}$




**Sine rule** $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

**Cosine rule** $a^2 = b^2 + c^2 - 2bc\cos A$

**Area** $T = \dfrac{ab\sin C}{2}$

**Trigonometric formulas**

$\sin^2 v + \cos^2 v = 1$

$\sin(u+v) = \sin u\cos v + \cos u\sin v$

$\sin(u-v) = \sin u\cos v - \cos u\sin v$

$\cos(u+v) = \cos u\cos v - \sin u\sin v$

$\cos(u-v) = \cos u\cos v + \sin u\sin v$

$\sin 2v = 2\sin v\cos v$

$$\cos 2v = \begin{cases} \cos^2 v - \sin^2 v & (1) \\ 2\cos^2 v - 1 & (2) \\ 1 - 2\sin^2 v & (3) \end{cases}$$

$a\sin x + b\cos x = c\sin(x+v)$ where $c = \sqrt{a^2+b^2}$ and $\tan v = \dfrac{b}{a}$

**Values of trigonometric functions**

| Angle $v$ (degrees) (radians) | $0°$ <br> $0$ | $30°$ <br> $\frac{\pi}{6}$ | $45°$ <br> $\frac{\pi}{4}$ | $60°$ <br> $\frac{\pi}{3}$ | $90°$ <br> $\frac{\pi}{2}$ | $120°$ <br> $\frac{2\pi}{3}$ | $135°$ <br> $\frac{3\pi}{4}$ | $150°$ <br> $\frac{5\pi}{6}$ | $180°$ <br> $\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $\sin v$ | $0$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | $1$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | $0$ |
| $\cos v$ | $1$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{\sqrt{3}}{2}$ | $-1$ |
| $\tan v$ | $0$ | $\frac{1}{\sqrt{3}}$ | $1$ | $\sqrt{3}$ | Not def. | $-\sqrt{3}$ | $-1$ | $-\frac{1}{\sqrt{3}}$ | $0$ |